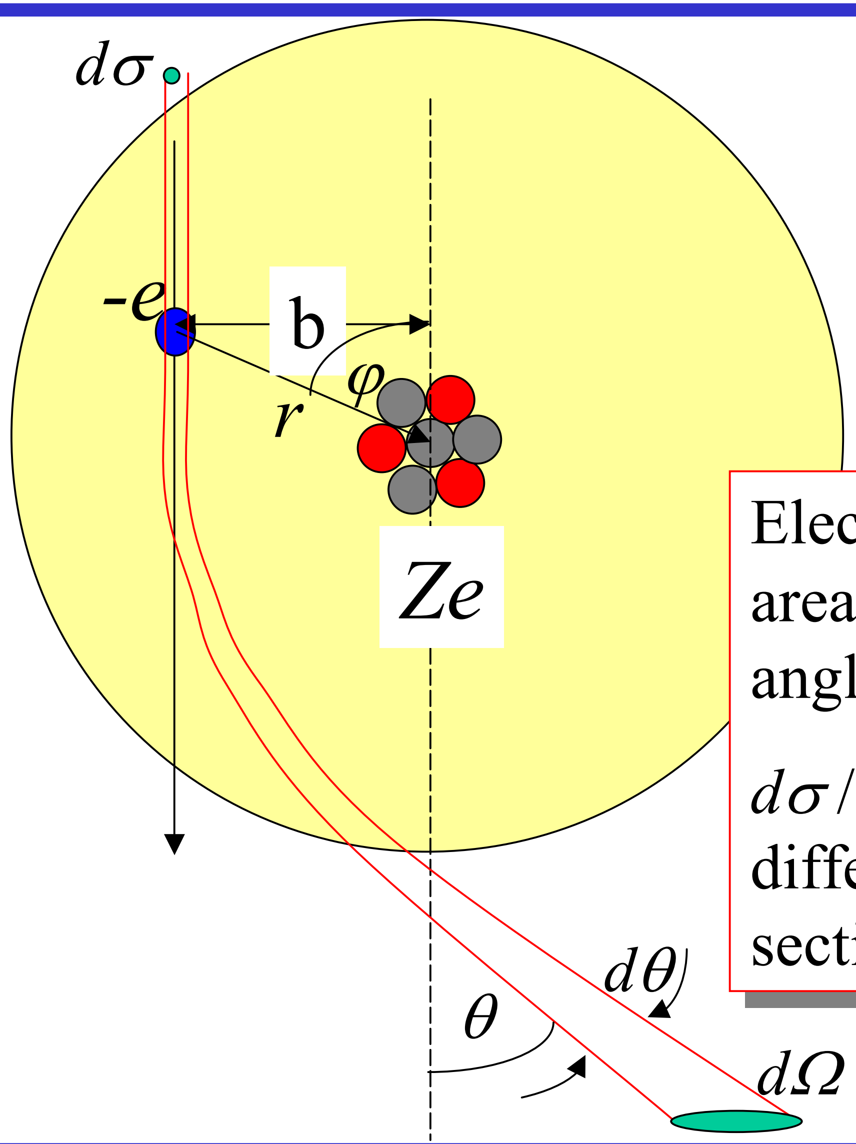
Electron-Solid Interactions

- Where do they go?
  - Elastic scattering
    - Single scattering
    - Plural scattering
    - Continuous slowing down approximation
    - Multiple scattering/diffusion
- What do they do?
  - Inelastic scattering
    - Energy loss mechanisms

Ludwig Reimer, *"Scanning Electron Microscopy"*, Springer-Verlag (1985)
Ludwig Reimer, *"Transmission Electron Microscopy"*, 4^{th}, Springer-Verlag (1997)

Elastic Scattering – Rutherford Cross-section



$$F = -\frac{e^2 Z}{4\pi\varepsilon_0 r^2}\mathbf{u}_r$$

Electrons that pass through the area $d\sigma$ are scattered through the angle θ into a solid angle $d\Omega$.

$d\sigma / d\Omega$ is referred to as the differential scattering cross section.

Rutherford Differential Scattering Cross-Section

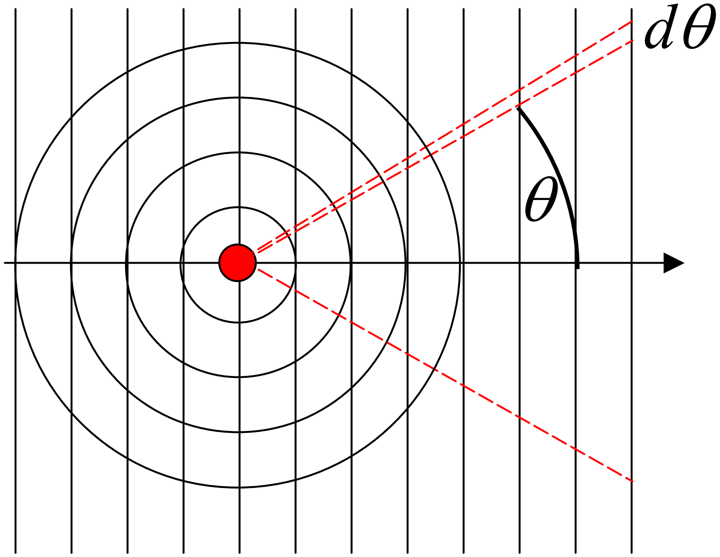
Consider angular momentum, resolve motion into horizontal and vertical components. At the start of the trajectory v_h is zero, at the end $v_h = v\sin\theta$:

$$\frac{d\sigma}{d\Omega} = \frac{e^4 Z^2}{4(4\pi\varepsilon_0)^2 m^2 v^4} \frac{1}{\sin^4(\theta/2)}$$

- Scattering proportional to Z^2
- Scattering forward peaked
  - Singularity at $\theta = 0$

Elastic Scattering – Other Cross-Sections

- Need to account for screening of nuclear charge by atomic electrons
  - Quantum mechanical treatment considers superposition of plane wave and spherical scattered wave



$$Plane\ wave: \quad \psi = \psi_0 e^{2\pi i k_0 z}$$

$$Scattered\ wave: \quad \psi = \psi_0 f(\theta) \frac{e^{2\pi i k_0 z}}{r}$$

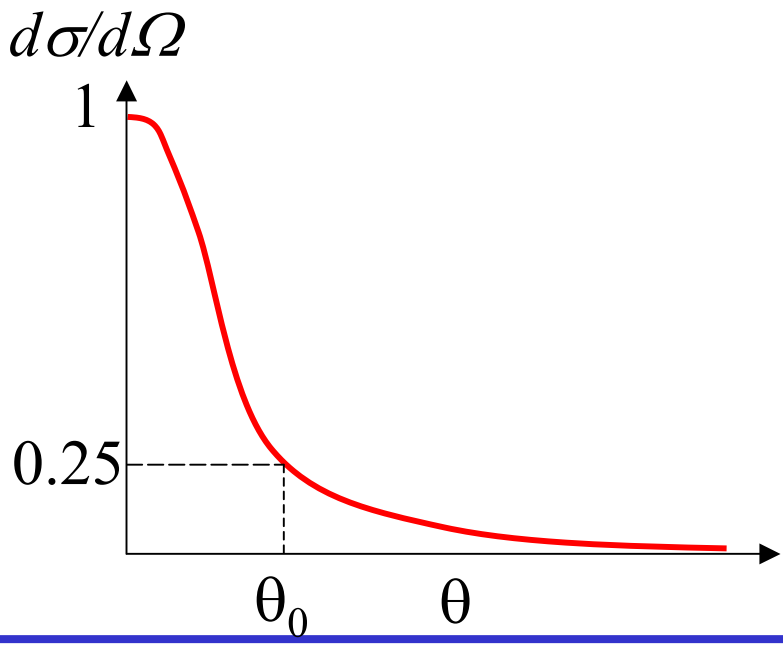
$$\frac{d\sigma}{d\Omega} = \left| f(\theta) \right|^2$$

- *f(θ)* is the angle dependent scattering amplitude and represents Fraunhofer (far-field) diffraction by the atomic potential
- Numerous cross-sections derived according to form of potential

Screened-Rutherford Cross-Section

$$\frac{d\sigma}{d\Omega} = \frac{4Z^2(1+E/E_0)}{a_H^2}\frac{1}{\left[1+(\theta/\theta_0)^2\right]^2}, \theta_0 = \frac{\lambda Z^{1/3}}{2\pi a_H}$$

θ_0 = characteristic scattering angle, typically 10's mrad at 100 kV



Total Elastic Cross-Section

$$\boxed{\sigma_{elastic} = \frac{Z^{4/3}\lambda^2(1+E/E_0)^2}{\pi}}$$

Cross-section *increases* as E *decreases*
Characteristic angle *increases* as E *decreases*

Mean-Free-Path

$N = atoms/unit\ volume$

$n = atoms/unit\ area$

$n\sigma = scattering\ area\ fraction$

$N_A = Avogadro's\ number$

$A = Atomic\ number$

$\rho = density$

$$n = \frac{N_A \rho}{A} t = Nt$$

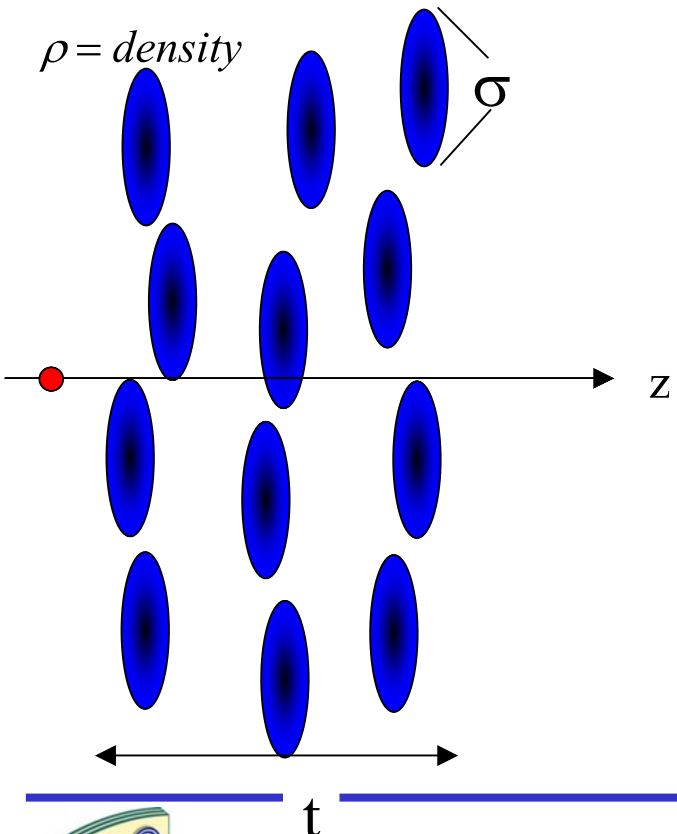
$$dI = -IN\sigma dz$$

$$I = I_0 e^{-N\sigma t} = I_0 e^{-t/\Lambda}$$

$$\Lambda = \frac{1}{N\sigma} = Mean\ Free\ Path$$

$$P(m) = \frac{(t/\Lambda)^m}{m!} e^{-t/\Lambda}$$

$$P(0) = e^{-t/\Lambda} = fraction unscattered$$



Element	Z	$\Lambda_{50\ kV}$ (nm)	Range (μm)
C	6	83	22.6
Al	13	49	16.7
Cu	29	10.7	5.1
Ag	47	7.7	4.3
Au	79	4.6	2.3

Angular Distribution & Beam Broadening

$$I_1(\theta)d\Omega = I_0 n \frac{d\sigma}{d\Omega} d\Omega = I_0 N\sigma t\left(\frac{1}{\sigma}\frac{d\sigma}{d\Omega}\right) d\Omega = I_0 \frac{t}{\Lambda} S_1(\theta) d\Omega,\ d\Omega = 2\pi \sin\theta$$

$$\int S_1(\theta) 2\pi \sin\theta d\theta = 1$$ ⇐ Normalized single scattering function

$$S_m = S_{m-1}(\theta) \otimes S_1(\theta)$$

$$I(\theta)d\Omega = I_0 d\Omega \sum_{m=0}^{\infty} \frac{(t/\Lambda)^m}{m!} e^{-t/\Lambda} S_m(\theta)$$

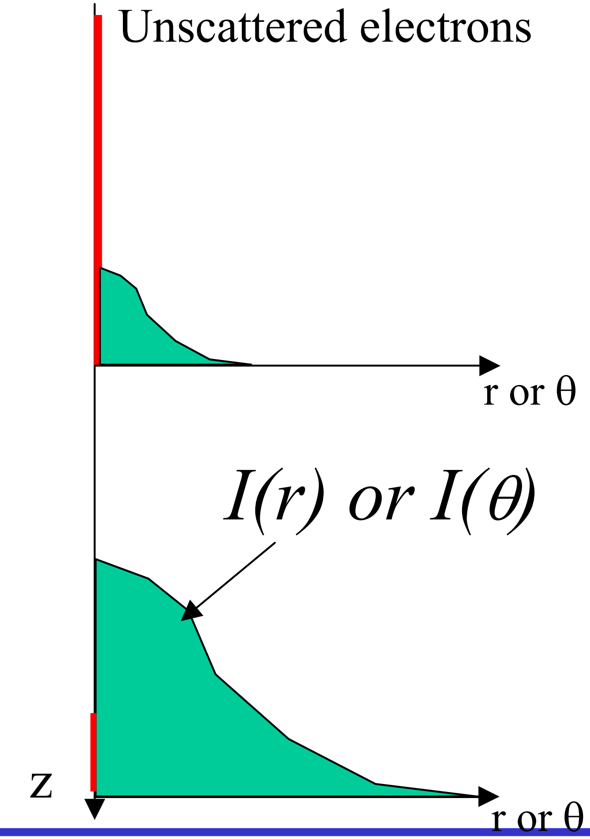
$$\overline{\theta^2} = \frac{t}{\Lambda}\overline{\theta_1^2}, \quad r = \theta z$$

Gaussian

$$Approximate\ S_1(\theta) \approx e^{-\theta^2/\overline{\theta_1^2}},\ assume \quad I(r) \propto e^{-r^2/\overline{r^2}}$$

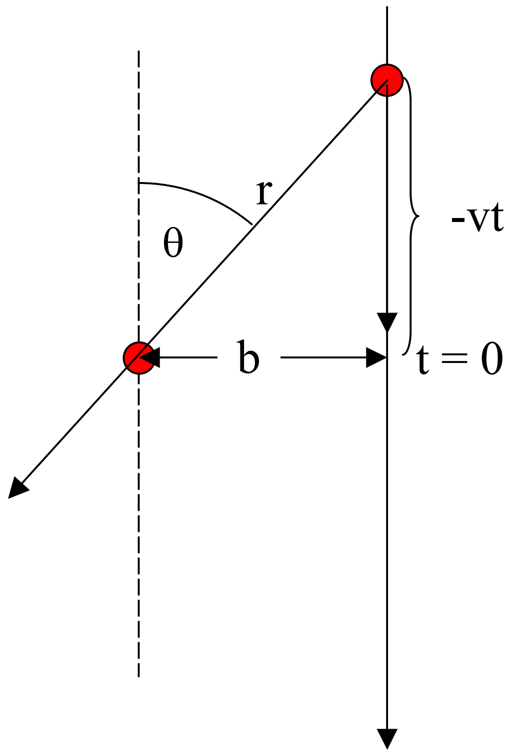
$$\overline{r^2} = \frac{2}{3}\frac{\overline{\theta_1^2}}{\Lambda} t^3, \quad r_{rms} \propto t^{3/2}$$

Appropriate for m< 25



Continuous Slowing Down Approximation

- Energy transfer, W ($\ll E$, incident electron energy), occurs through Coulomb interaction between incident electrons and atomic electrons. Mean energy loss/path length is $-dE_m$. Stopping power, S:

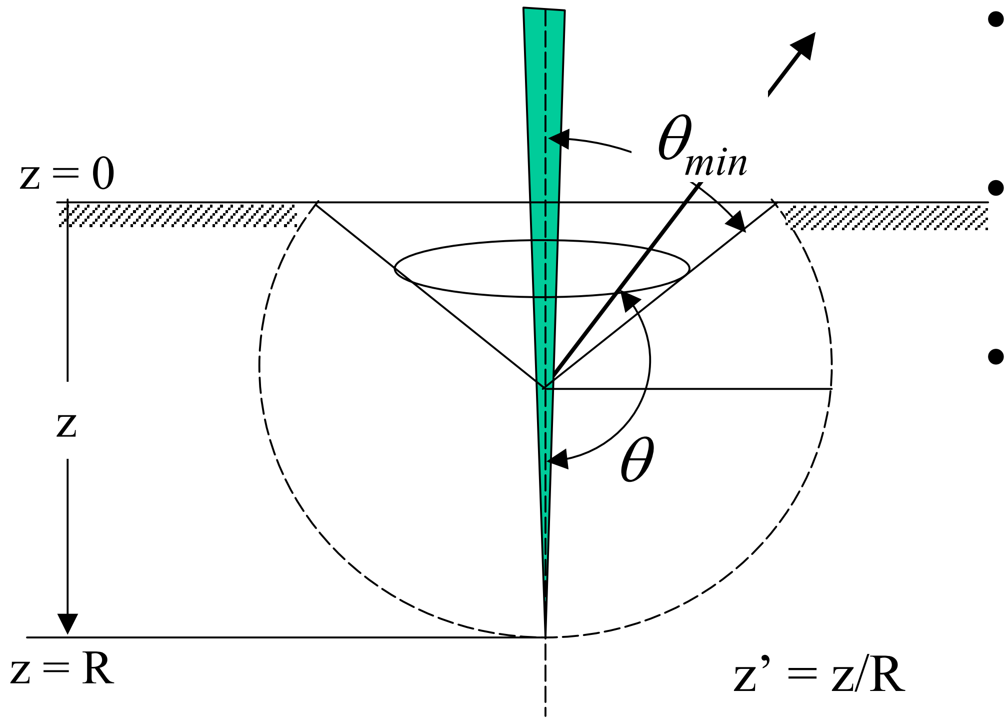


$$S = \left|\frac{dE_m}{ds}\right| = NZ\int_{W_{\min}}^{W_{\max}} \frac{d\sigma}{dW} W\,dW = \frac{2\pi e^4 N_A \rho Z}{(4\pi\varepsilon_0)^2 AE}\int_{b_{\min}}^{b_{\max}} \frac{db}{b}$$

$$= \frac{2\pi e^4 N_A \rho Z}{(4\pi\varepsilon_0)^2 AE}\ln\left(\frac{E}{J}\right), \quad J = 11.5Z, \quad Z \le 6$$

$$Range,\ R \propto E^n, \frac{1}{Z^m}, \quad 1.3 \le n \le 1.7,\ m \approx 1$$

Diffusion - Everhart's Single Scattering Model

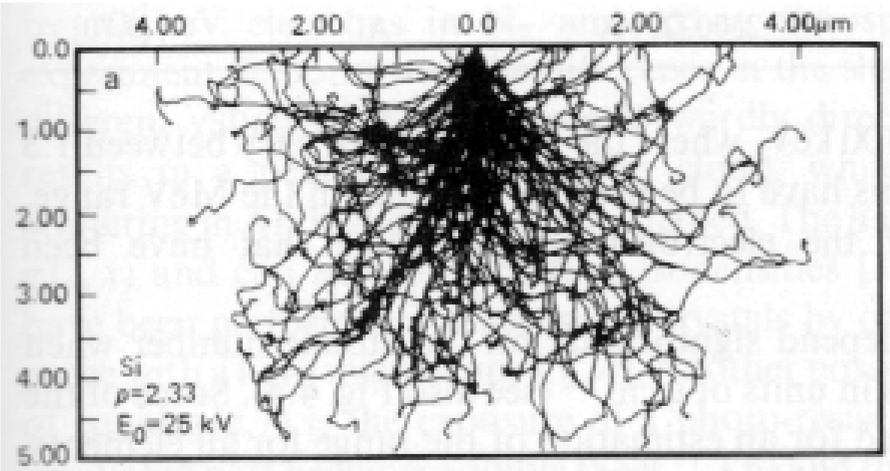


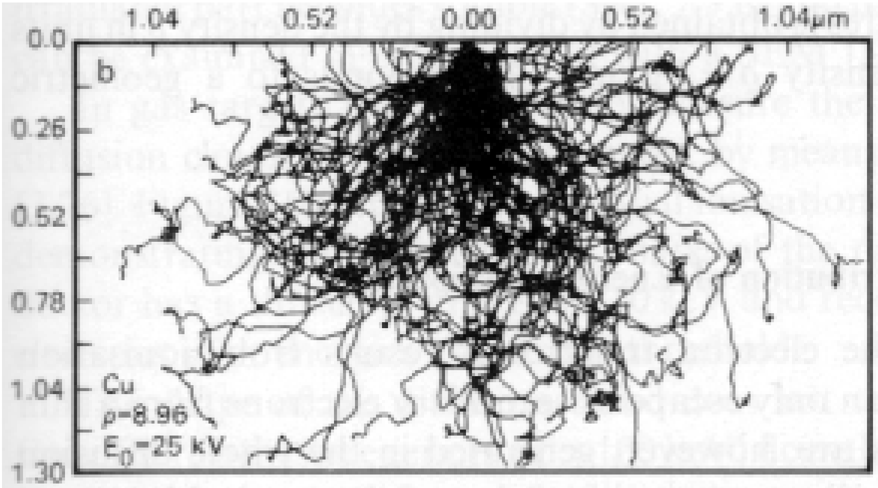
- Electron energy decreases with depth as v = $(c_T \rho z - v_0^{\,4})^{1/4}$
- Intensity decreases as $dI(z) = N_A \rho \sigma \pi/(2A)\, I(z) dz$
- Electrons are backscattered by single scattering through angles $\pi - \theta_{min} < \theta < \pi$

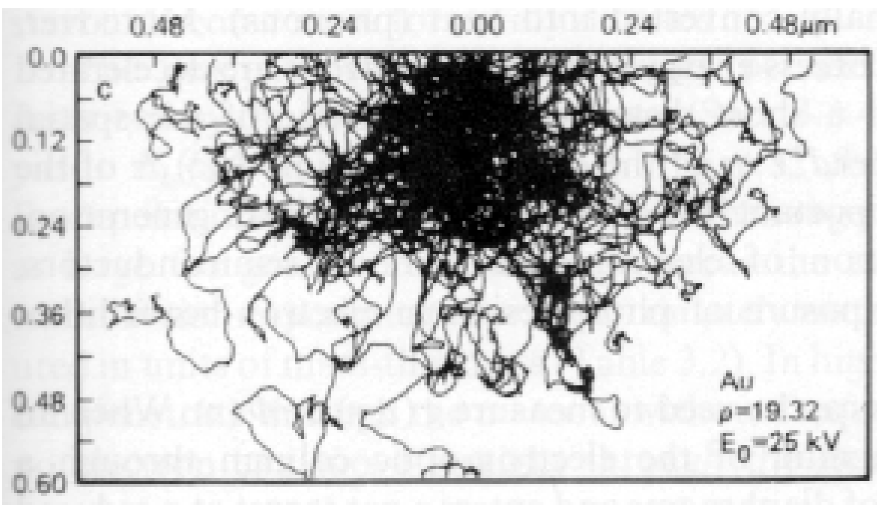
$$\eta = \int_0^{0.5} \frac{N_A \rho}{A I_0} \sigma(\theta_{\min}) I(z') dz' \approx \frac{0.012Z - 1 + 0.5^{0.012Z}}{0.012Z + 1}$$

η increases with increasing Z, R decreases with increasing Z. Character of proximity effect changes with atomic number

Monte-Carlo Simulation

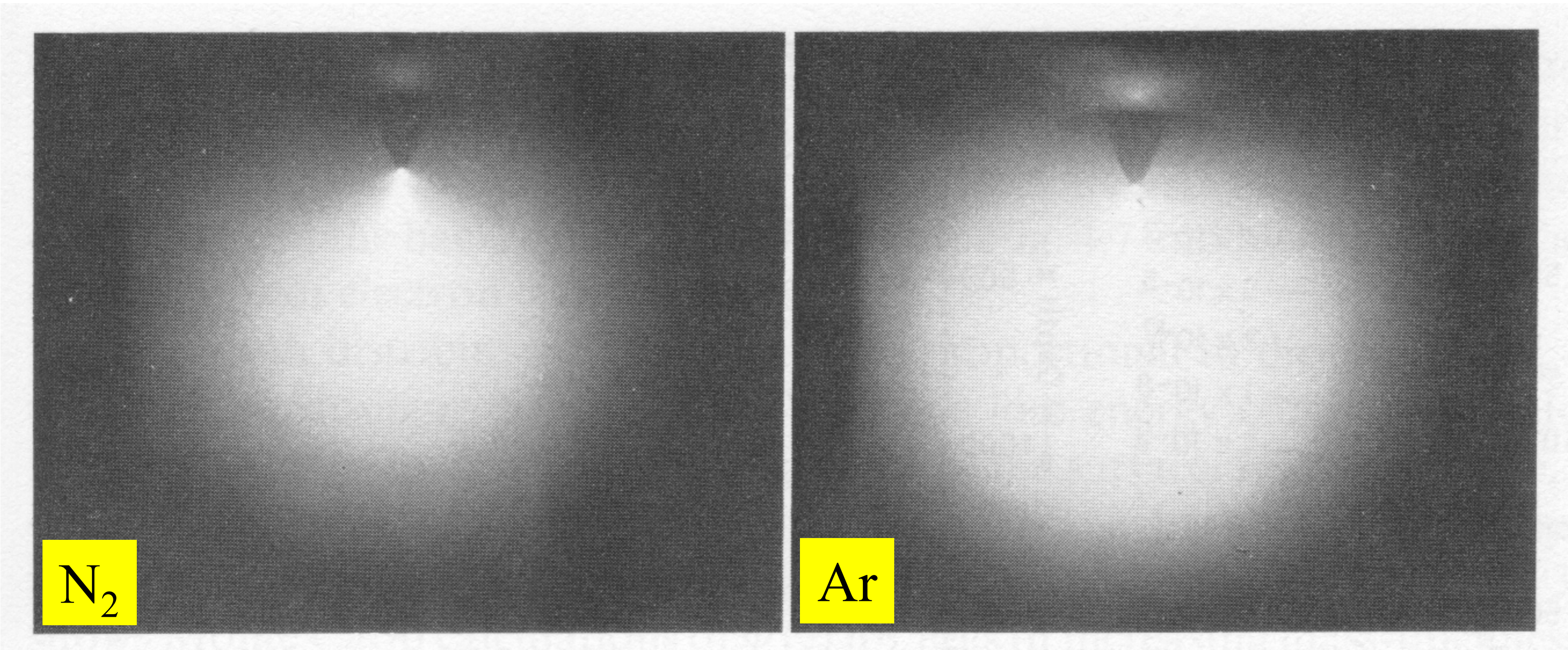






Note changes in horizontal and vertical scales as the atomic number increases: Si 14, Cu 29, Au 79

Scattering in Gas Targets



Ludwig Reimer, *"Scanning Electron Microscopy"*, Springer-Verlag (1985)

Inelastic Scattering

- Energy loss occurs through a variety of mechanisms
  - Molecular oscillations/phonons
    - ΔE = 20 meV - 1 eV
  - Conduction/valence electrons
    - Plasmons
    - Inter- or Intra-band transitions
    - ΔE = 1 eV - 50 eV
  - Core electrons
    - Ionization of inner shell electrons
      - X-rays
      - Auger electrons
    - ΔE_K = 110 eV (Be) - 80 keV (Au)

Described by dielectric theory - related to optical constants of material. Electron energy-loss spectra and those for light and x-rays are related

Energy loss in C is ≈ 0.24 eV/nm at 100 keV

Plasmon & Optical Losses I

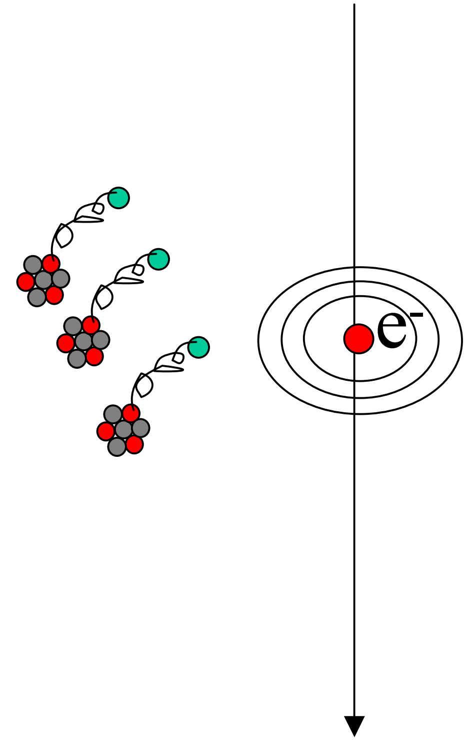
$$m^* \frac{d^2\mathbf{x}}{dt^2} + m^*\gamma \frac{d\mathbf{x}}{dt} = -e\mathbf{E}$$

$$\mathbf{x} = \frac{e}{m^*\omega^2}\frac{\omega^2 - i\omega\gamma}{\omega^2 + \gamma^2}\mathbf{E}$$

$$\mathbf{P} = -eN_e\mathbf{x} = \varepsilon_0\chi_e\mathbf{E}, \quad \varepsilon = \varepsilon_0(1 + \chi_e)$$

$$\varepsilon(\omega) = \varepsilon_1 + i\varepsilon_2 = \varepsilon_0\left(1 - \frac{N_e e^2}{m^*\varepsilon_0}\frac{1}{\omega^2 + i\omega\gamma}\right)$$

$$\Rightarrow \varepsilon_1 = \varepsilon_0\left(1 - \frac{\omega_{pl}^2}{\omega^2}\frac{1}{1+(\gamma/\omega)^2}\right), \quad \varepsilon_2 = \varepsilon_0\frac{\gamma}{\omega}\frac{\omega_{pl}^2}{\omega^2}\frac{1}{1+(\gamma/\omega)^2}$$



$$\textit{Plasma frequency}: \omega_{pl} = \sqrt{\frac{N_e e^2}{\varepsilon_0 m^*}} \quad i.e.\ \gamma = 0, \quad \textit{Plasmon energy}: \quad \Delta E_{pl} = \hbar\omega_{pl}$$

Plasmon & Optical Losses II

$$m^*\left(\frac{d^2\mathbf{x}}{dt^2}+\gamma\frac{d\mathbf{x}}{dt}+\omega_b^2\mathbf{x}\right)=-e\mathbf{E}$$

$$\varepsilon(\omega)=\varepsilon_0\left(1+\frac{N_e e^2}{\varepsilon_0 m^*}\frac{1}{\omega_b^2-\omega^2-i\omega\gamma}\right)$$

Introduce oscillators with other characteristic frequencies,ω_b, to represent bound electrons. Resonances occur when $\omega=\omega_b$. Passage of high-energy electron results in frequency pulse that can excite many resonances.

Lorentz oscillators: http://webphysics.davidson.edu/Projects/AnAntonelli/myThesis.html

Dielectric Theory

Optical constant : $\varepsilon = \varepsilon_1 + i\varepsilon_2 = (n + ik)^2$

Energy dissipation : $\dfrac{dW}{dt} = \dot{\mathbf{E}}.\mathbf{D}$

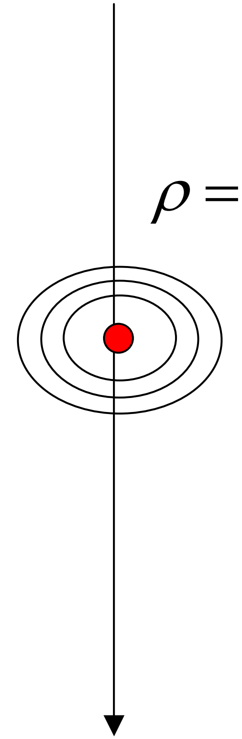
Electron : $\rho = e\delta(x - vt)$

$div\mathbf{D} = \rho, \quad \mathbf{E} = \mathbf{E}_0 e^{-i\omega t}, \quad \mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$

$$\Rightarrow \overline{\frac{dW}{dt}} = \varepsilon_0\varepsilon_2 \frac{\omega|\mathbf{E}_0|^2}{2} = \frac{1}{\varepsilon_0}\frac{\varepsilon_2}{|\varepsilon|^2}\frac{\omega D_0{}^2}{2} = \frac{1}{\varepsilon_0}\frac{\omega D_0{}^2}{2}\mathrm{Im}(-1/\varepsilon)$$

$Note: \quad \varepsilon \to f(\omega)$

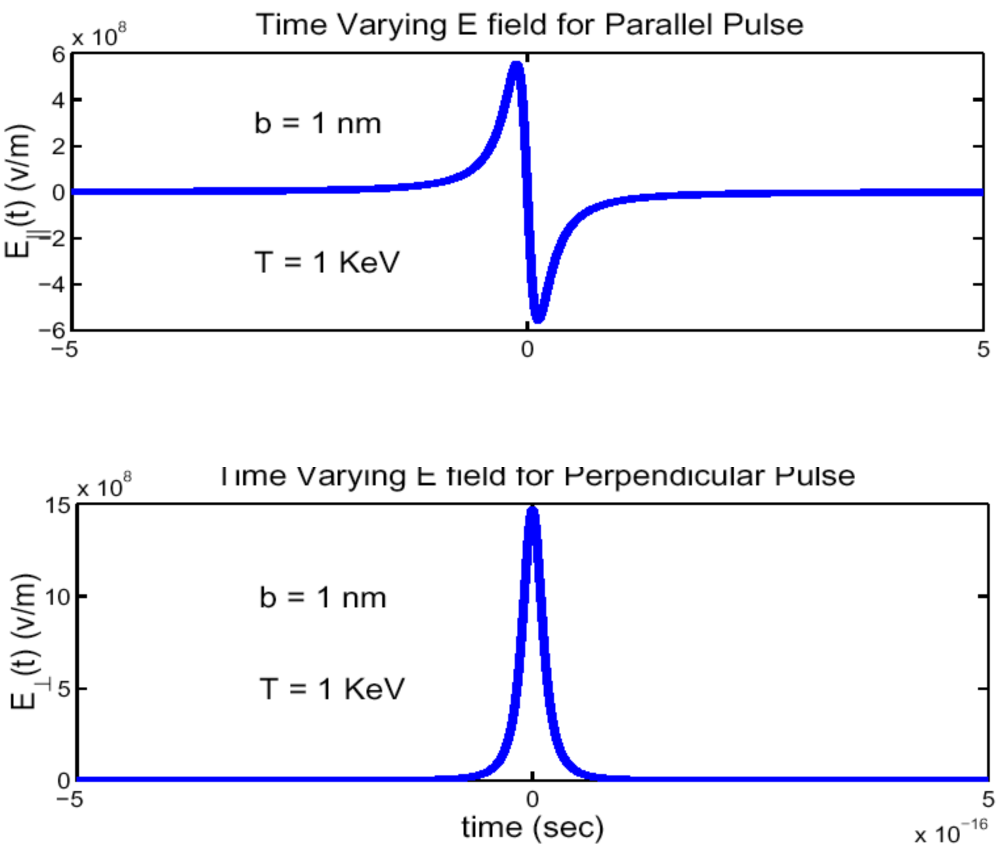
$$\frac{d^2\sigma}{d\Delta E d\Omega} = \frac{1}{\pi^2 a_H m v^2 N_e}\frac{\mathrm{Im}(-1/\varepsilon)}{(\theta^2 + \theta_E^2)}, \quad N_e = electrons/unit\ volume, \quad \theta_E = \frac{\Delta E}{2E}$$

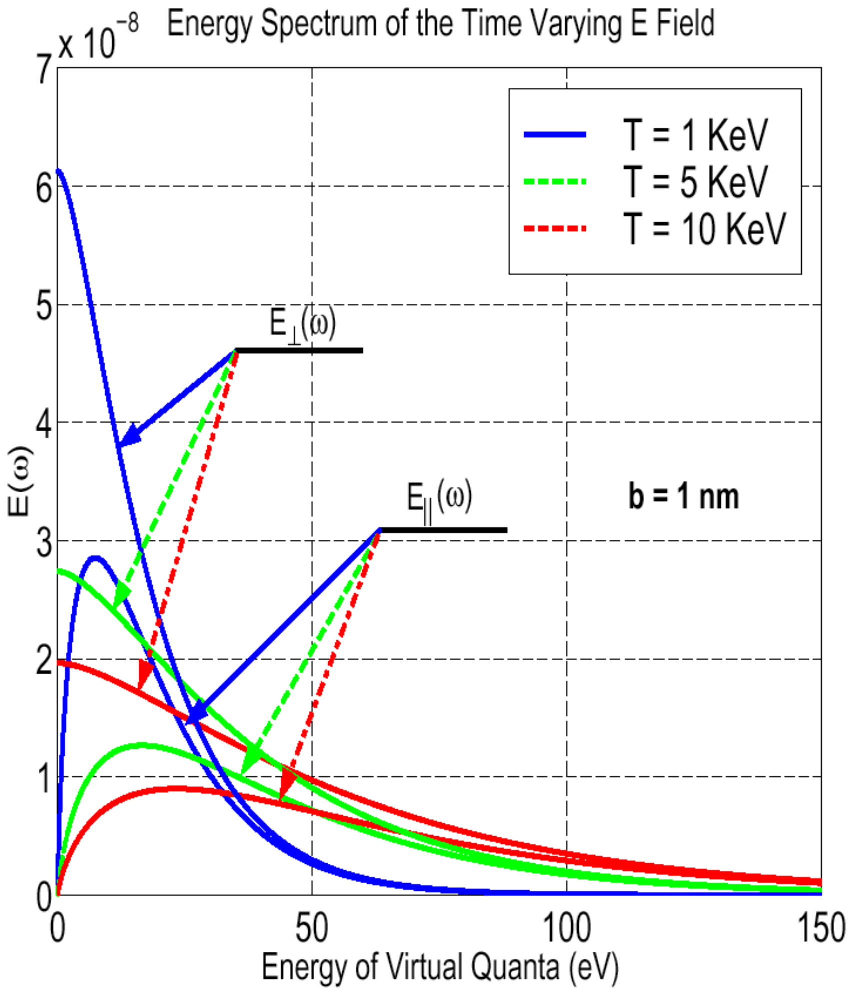


$\sigma_{inel} \approx 20\ \sigma_{el}/Z$

20 eV loss at 100 keV gives a θ_E of 0.1 mrad

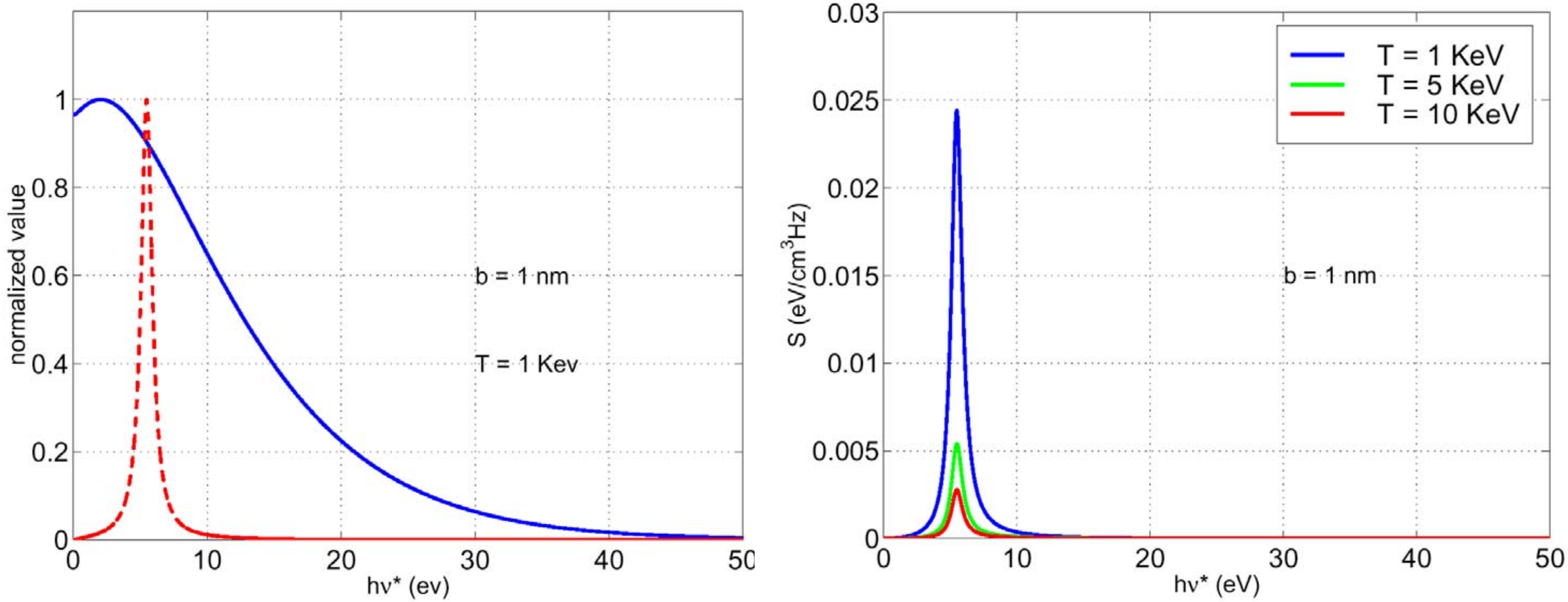
Method of Virtual Quanta





"Energy Transfer Between Electrons and Photoresist: Its Relation to Resolution", Geng Han and Franco Cerrina, J. Vac. Sci. Technol. B18 p3297 (2000)

Energy Transfer



Electrons become bluer and dimmer as their energy increases

"Energy Transfer Between Electrons and Photoresist: Its Relation to Resolution", Geng Han and Franco Cerrina, J. Vac. Sci. Technol. B18 p3297 (2000)

EELS Spectrum (SiN)